Arius®

Tail Factor Analysis

Milliman

IT TAKES VISION

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1. Tail Factor Analysis

Arius provides a handy tool to help you estimate tail factors for your development exhibits. The Tail Factor Analysis (TFA) tool fits three types of curves — inverse power, exponential, and Weibull — and uses the resulting formulas to help project future development periods.

This tool allows you to:

- select a series of development factors to be the basis for the tail factor analysis,
- fit several different curves to those factors, and extend the curves any number of periods into the future,
- use the fitted cumulative development factors to provide an estimated tail factor, and
- add these fitted factors and tail factor to your exhibit, from which you can select development factors.

The Tail Factor Analysis tool also provides a number of options to help you obtain the most reasonable estimate possible.

- Specify the point in time at which to assume no further development (development factor = 1.0).
- Exclude or modify specific factors, and see any effects on your regression.
- See the equation of the line and the R^2 value of the fit. (The R^2 value represents the reliability of the regression, where values closer to 1.0 indicate a stronger correlation.) These amounts are updated continuously as you modify the selected data.
- Work with a graphical view of your data and the fitted line, including the ability to zoom in on the graph to see more detail.

2. The Curves

The curve formulas provided in Arius exhibit patterns of development found in many property and casualty lines of business. Each curve has somewhat different properties. You will often find that at least one of the formulas provides a curve that resembles the tendencies in your data.

Arius provides three curve formulas:

- inverse power curve
- exponential curve
- Weibull curve

INVERSE POWER CURVE

Using the development factors you choose, Arius calculates a line of fitted factors using the form

$$y = 1.0 + e^{a}(x + c)^{-b}$$

Specifically, it:

- begins with **x** and **y** coordinates as follows:
- **x** coordinates: an index based on each selected factor's respective age, plus a constant factor to adjust the starting point of the index for a potentially better fit
- **y** coordinates: incremental development factors
- takes the natural logarithm of **1/(x + c)** and **y-1** in order to perform a linear regression

  $y = 1.0 + e^{a}(x + c)^{-b}$
  $\ln (y-1) = a + b (\ln(1/(x + c)))$
  $y' = a + b (x + c)'$

  Note that **y**, the incremental development factor, must be greater than 1.0 to calculate the logarithm.
- performs a linear regression of **$y' = a + b(x + c)'$** to estimate values for **a** and **b**
- plugs **a** and **b** back into the formula **$y = 1.0 + e^{a}(x + c)^{-b}$** to derive the fitted curve

EXPONENTIAL CURVE

Using the development factors you choose, Arius calculates a line of fitted factors using the form

$$y = 1.0 + e^{a+bx}$$

Specifically, it:

- begins with **x** and **y** coordinates as follows:
- **x** coordinates: an index based on each selected factor's respective age
- **y** coordinates: incremental development factors

- takes the natural logarithm of **1/x** and **y-1** in order to perform a linear regression

  $y = 1.0 + e^{a+bx}$
  $\ln (y-1) = bx$
  $y' = a + bx'$

  Note that **y**, the incremental development factor, must be greater than 1.0 to calculate the logarithm

- performs a linear regression of **y' = a + bx'** to estimate values for **a** and **b**
- plugs **a** and **b** back into the formula **$y = 1.0 + e^{a+bx}$** to derive the fitted curve

WEIBULL CURVE

Using the development factors you choose, Arius calculates a line of fitted factors using the form

$y = 1.0/(1-e^{(-e^a(x + c)^b)})$

Specifically, it:

- begins with **x** and **y** coordinates as follows:
- **x** coordinates: an index based on each selected factor's respective age, plus a constant factor to adjust the starting point of the index for a potentially better fit
- **y** coordinates: incremental development factors
- takes the natural logarithm of **1/(x + c) and y-1** in order to perform a linear regression

  $y = 1.0/(1-e^{(-e^a(x + c)^b)})$
  $\ln (1-1/y) = -e^a (x + c)^b$
  $y' = a + b(x + c)'$

  Note that **y**, the incremental development factor, must be greater than 1.0 to calculate the logarithm.

- performs a linear regression of **y' = a + b(x + c)'** to estimate values for **a** and **b**
- plugs **a** and **b** back into the formula

  $y = 1.0/(1-e^{(-e^a(x + c)^b)})$ to derive the fitted curve

FORMULAS FOR FACTORS LESS THAN 1

In certain situations the development factors being fit can be less than 1, curving asymptotically up to 1.0 rather than all greater than 1 and curving down to 1.0. Arius provides alternative equations to fit these different asymptotic curves.

Simply select the check box for **Fit values less than 1** and the system will use these alternate formulas:

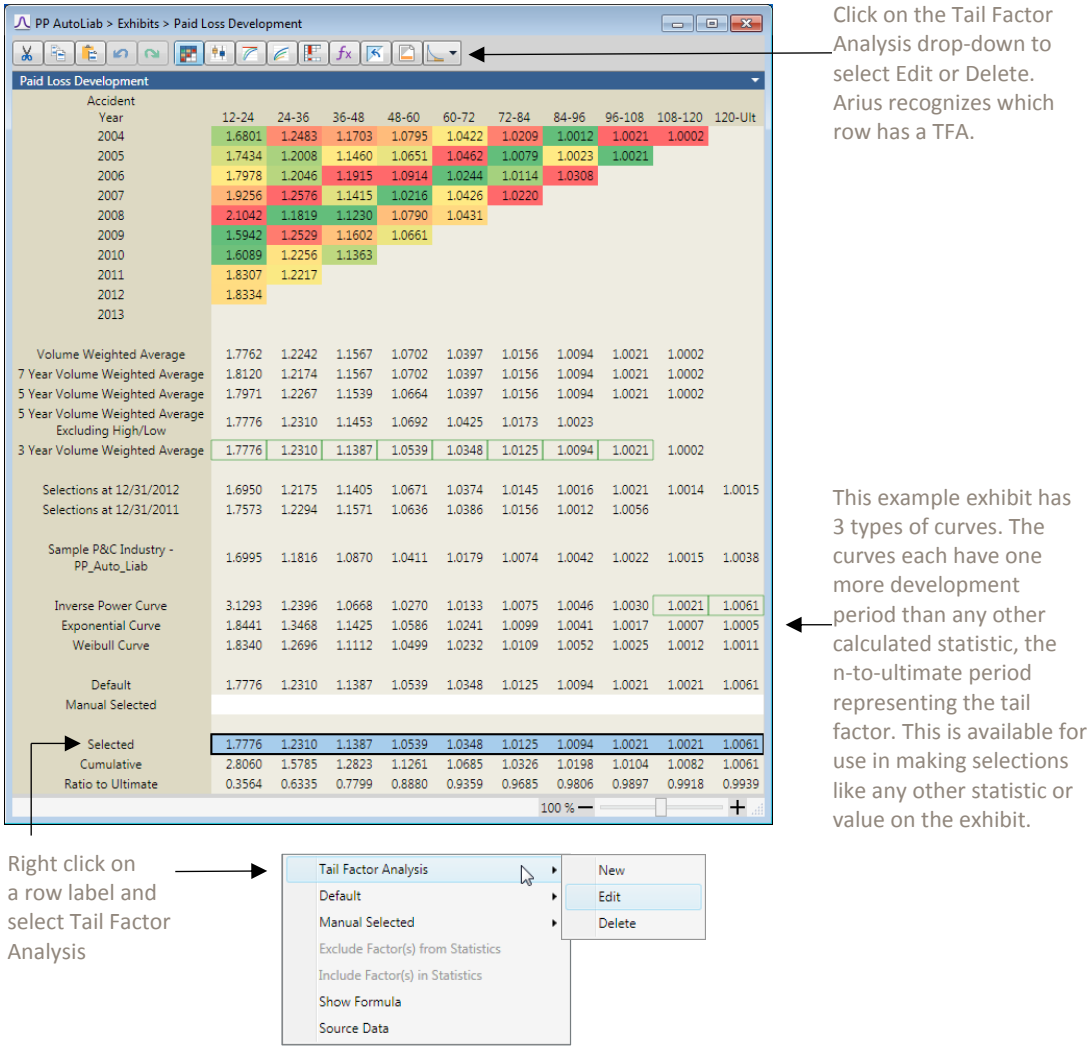
Inverse power curve	$y = 1.0 - e^a(x + c)^{-b}$
Exponential curve	$y = 1.0 – e^{(a+bx)}$
Weibull curve	$y = 1-e^{(-e^a(x + c)^b)}$

3. Working with the TFA tool

ADD, EDIT, OR DELETE A TAIL FACTOR ANALYSIS

1. Open the exhibit you plan to work with.
2. Click on a row label to select a row of data to which you would like to fit a curve. This will often be your Selected row after you have made initial factor selections.
3. Click the drop-down menu from the **Tail Factor Analysis** icon on the exhibit's ribbon and select **New**, **Edit** or **Delete**. Or right-click on the row label (from step 2 above) and select Tail Factor Analysis, then New, Edit, or Delete from the pop-up menu.

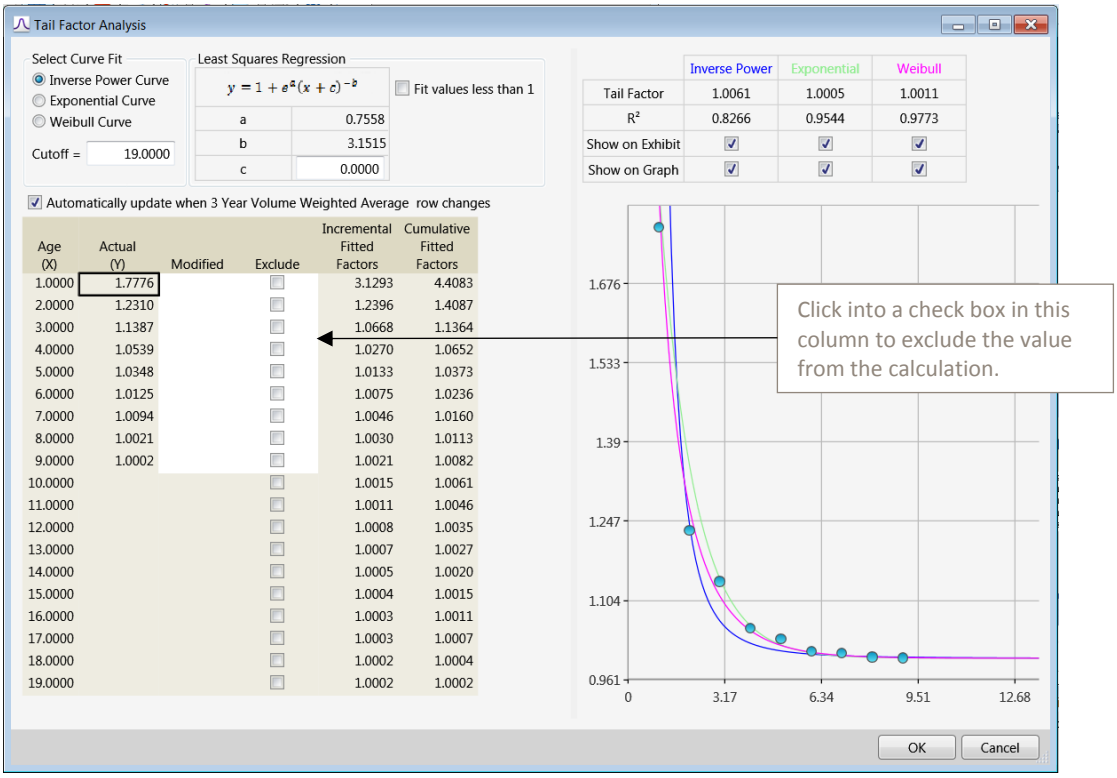
PAID LOSS DEVELOPMENT EXHIBIT



Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
2004	1.6801	1.2483	1.1703	1.0795	1.0422	1.0209	1.0012	1.0021	1.0002	
2005	1.7434	1.2008	1.1460	1.0651	1.0462	1.0079	1.0023	1.0021		
2006	1.7978	1.2046	1.1915	1.0914	1.0244	1.0114	1.0308			
2007	1.9256	1.2576	1.1415	1.0216	1.0426	1.0220				
2008	2.1042	1.1819	1.1230	1.0790	1.0431					
2009	1.5942	1.2529	1.1602	1.0661						
2010	1.6089	1.2256	1.1363							
2011	1.8307	1.2217								
2012	1.8334									
2013										
Volume Weighted Average	1.7762	1.2242	1.1567	1.0702	1.0397	1.0156	1.0094	1.0021	1.0002	
7 Year Volume Weighted Average	1.8120	1.2174	1.1567	1.0702	1.0397	1.0156	1.0094	1.0021	1.0002	
5 Year Volume Weighted Average	1.7971	1.2267	1.1539	1.0664	1.0397	1.0156	1.0094	1.0021	1.0002	
5 Year Volume Weighted Average Excluding High/Low	1.7776	1.2310	1.1453	1.0692	1.0425	1.0173	1.0023			
3 Year Volume Weighted Average	1.7776	1.2310	1.1387	1.0539	1.0348	1.0125	1.0094	1.0021	1.0002	
Selections at 12/31/2012	1.6950	1.2175	1.1405	1.0671	1.0374	1.0145	1.0016	1.0021	1.0014	1.0015
Selections at 12/31/2011	1.7573	1.2294	1.1571	1.0636	1.0386	1.0156	1.0012	1.0056		
Sample P&C Industry - PP_Auto_Liab	1.6995	1.1816	1.0870	1.0411	1.0179	1.0074	1.0042	1.0022	1.0015	1.0038
Inverse Power Curve	3.1293	1.2396	1.0668	1.0270	1.0133	1.0075	1.0046	1.0030	1.0021	1.0061
Exponential Curve	1.8441	1.3468	1.1425	1.0586	1.0241	1.0099	1.0041	1.0017	1.0007	1.0005
Weibull Curve	1.8340	1.2696	1.1112	1.0499	1.0232	1.0109	1.0052	1.0025	1.0012	1.0011
Default	1.7776	1.2310	1.1387	1.0539	1.0348	1.0125	1.0094	1.0021	1.0021	1.0061
Manual Selected										
Selected	1.7776	1.2310	1.1387	1.0539	1.0348	1.0125	1.0094	1.0021	1.0021	1.0061
Cumulative	2.8060	1.5785	1.2823	1.1261	1.0685	1.0326	1.0198	1.0104	1.0082	1.0061
Ratio to Ultimate	0.3564	0.6335	0.7799	0.8880	0.9359	0.9685	0.9806	0.9897	0.9918	0.9939

4. Selecting **New** or **Edit** opens the TFA module. The factors you selected display as the **Y-axis** values. The **X-axis** values are based on the selected factors' respective ages. The curve defaults to ten periods beyond the last period of actual data.

TFA WINDOW



1. Choose the curve you want to work with by selecting the **Inverse Power, Exponential,** or **Weibull** radio button.
2. Determine how far out into the future the development periods should be estimated. Adjust the Cutoff field accordingly. This will often be an estimate of the number of periods required for full development on this book or line of business.
3. Determine whether any of your data points should be excluded from the calculation to provide an equation that better fits your data.
   - Click any checkbox under the **Exclude** column to exclude that point from the calculation. Alternatively, you can also click on any dot on the graph to exclude a point.
   - For example, you can often obtain a higher R^2 value by eliminating the first value, or any other point that appears to be a significant outlier. The graph can be an effective tool for identifying such unusual points.
4. You can adjust any data points used in your calculations by entering adjusted values in the Modified column. Amounts in this column override the displayed **Y-axis** values in the calculations.
5. The Inverse Power and Weibull curves provide a **c** parameter as a means of adjusting the **x** value. Depending on the data, sometimes adding 1 or 2 to the **x** value through this parameter can provide a better fit.
6. When you're satisfied with the calculations, click **OK** to return to the development exhibit in Arius. Any or all of the three new statistics, representing the incremental factors at each point along the calculated line, can be added to your exhibit.

DYNAMIC AND STATIC CURVE FITS

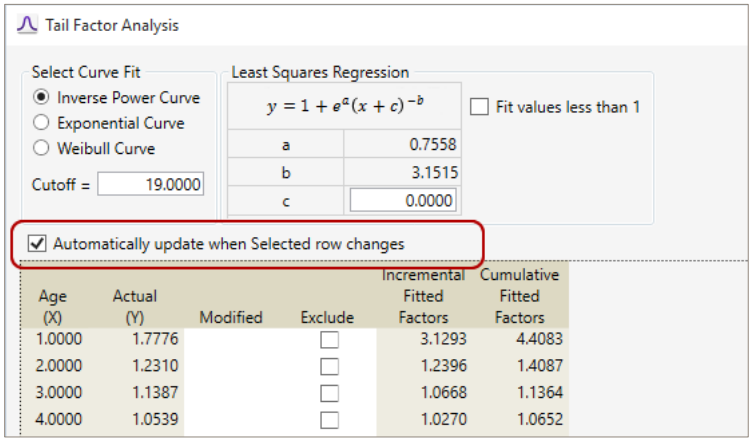
The fitted curves returned to your exhibit from the TFA module can be either static or dynamic. Specifically, you have the following options:

- Once fit to your selected factors or any other chosen row of statistics, they can retain their values on the exhibit even if the underlying data that was originally fit subsequently changes.
- Arius can dynamically update the values in the curve fit rows on your exhibit anytime there is a change to the data that was originally fit.

The dynamic recalculation can be helpful if you are using the curve's last value (*n* to Ultimate column) as the default for your selected tail, and you either update the underlying data in the triangle or add a new evaluation of data. In either scenario, the TFA would dynamically update the curve fits, and the default would provide a new selected tail, updating ultimate estimates from any methods relying on those factors as soon as your new data is loaded.

The static approach may be more helpful as a way to document how you made your tail selections without concern that the original calculations might be replaced with new data.

You can choose how to set this option by selecting the check box on the TFA Window:



4. TFA graphs

You can view the results of any or all of the formulas in the graph window.

- Use the check boxes at the top of the graph window to select which curves to display.
- Hold your mouse pointer over the point to show the exact coordinate of the data.
- Click on a point of the graph to exclude it from the calculation of the displayed curve and from the graph.
- Use the zoom feature for a closer view.

ZOOM

The Zoom feature lets you enlarge an area of your graph to see it in more detail. To enlarge a section of your graph, perform the following:

1. Right click on the graph and select **Enable Zoom**.
2. You can then use the controls in the top left corner to zoom in on a selected area, or to reset the zoom to the original level.

TAIL FACTOR ANALYSIS WINDOW