

Arius[®]

Interpolation and Extrapolation



IT TAKES VISION

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Table of Contents

- 1. INTRODUCTION 1
- 2. DETAILS BEHIND THE CALCULATIONS 2
 - Interpolation functions 2
 - Interpolation method 3
 - Sample interpolation procedure..... 4
- 3. USE CASE #1: INTERIM RESERVE ANALYSES 7
 - Prepare your project for interpolation 8
 - Define which interpolated results display on exhibits 10
 - Select interpolated development factors 11
 - Project method ultimates 12
- 4. USE CASE #2: INTERPOLATING HISTORICAL AND EXTERNAL FACTORS 13
- 5. USE CASE #3: INTERPOLATING PAYMENT PATTERNS 14

1. Introduction

One of the crucial roles of a reserving actuary is the selection of development factors. While the selection of factors is a straightforward process, the triangles' data structures require interpolating development factors in some situations. Also, other situations require extrapolation to account for development beyond the triangle.

The most common interpolation application within an Arius project is to derive relevant factors in a partial period scenario. For example, suppose you perform an annual analysis with a partial last diagonal. In that case, the loss development factors require interpolation to apply to the *to date* experience, which has a development age not consistent with the development ages at which you select the loss development factors.

Arius derives interpolated factors using piecewise interpolation between known points. For example, a piecewise linear interpolation connects a straight line between the two points and assumes development is proportional to time over the period. Arius provides the capability to use linear, exponential, inverse power, or Weibull functions for interpolation purposes. Arius also allows you to specify whether the loss development factors or the ratios to ultimate are used as the *known* points.

The algorithms also include the ability to extrapolate the development factors beyond the first and last known values (e.g., for ages < 12 and factors beyond the tail), adjusting for both maturity and incomplete exposure periods.

These algorithms are heavily relied upon throughout Arius for

- selected development factors (for interim reserve analyses),
- historical and external factors (for benchmark patterns), and
- payment patterns (for cash flow reporting).

This document provides details behind the interpolation algorithm and describes how it is applied within the Arius Deterministic module.

2. Details behind the calculations

The interpolation algorithm within Arius returns an array of new_y's for a series of new_x's by using existing arrays of known x's and known y's. For example, we walk through this technique and convert selected development patterns (known_y's) at 12-24-36-... months (known_x's) to a new interpolated development pattern (new_y's) at 3-15-27-... months (new_x's).

The notation we use is:

New_y's = **InterpolateFunction**(known_y's, known_x's, new_x's), where

- **Function** relates to the equation used to fit the two known data points provided. Arius provides functions for the Exponential, Inverse Power, Linear, and Weibull curves.
- **Known_y's** represent the array of y-values you already know. This pattern can be described in terms of age-to-ultimate (cumulative) factors or percent of ultimate (ratio to ultimate) values.
- **Known_x's** represent the array of x-values you already know, e.g., the development ages associated with the known_y's above {12-24-36...84}. This array is specified by your data structure (i.e., number of development periods, length of development periods, and starting period in months).
- **New_x's** represents the new array of x-values for which you want InterpolateFunction to return corresponding y-values, e.g., the development ages corresponding to the actual evaluation date of your latest diagonal {3-15-27...75}.

INTERPOLATION FUNCTIONS

Arius uses Exponential, Inverse Power, Linear, and Weibull functions for interpolation. Translated for running a linear regression and solving directly for the a and b parameters, the formulas in Arius are as follows (where y = actual loss development factor, x = age of development in months).

FUNCTION	REGRESSION EQUATION	LINEAR TRANSFORMATION
Linear	$y = a + bx$ for $y > 0$	$y = a + bx$ for $y > 0$
Exponential	$y = e^{a+bx}$ for $y > 0$	$\ln(y) = a + bx$ for $y > 0$
Inverse Power	$y = \begin{cases} (1 + e^a x^{-b})^{-1}, & \text{for } 0 < y < 1 \\ 1 + e^a x^{-b}, & \text{for } y > 1 \end{cases}$	$\ln\left(\frac{1}{y} - 1\right) = a + b * \ln\left(\frac{1}{x}\right)$ for $0 < y < 1$ $\ln(y - 1) = a + b * \ln\left(-\frac{1}{x}\right)$ for $y > 1$,
Weibull	$y = \begin{cases} 1 - \exp(-e^a x^b) & \text{for } 0 < y < 1 \\ (1 - \exp(-e^a x^b))^{-1} & \text{for } y > 1 \end{cases}$	$\ln(-\ln(1 - y)) = a + b * \ln(x)$, for $0 < y < 1$ $\ln(-\ln(1 - 1/y)) = a + b * \ln(x)$ for $y > 1$

Given two known data points, the first step is to find the slope (*b*) and intercept (*a*). We transform the above equations into a linear form, shown in the final column. Once *a* and *b* are derived, the values are plugged back into the original formulas along with the new_x to arrive at the interpolated value or new_y.

INTERPOLATION METHOD

Arius uses the Average Date of Loss Approximation method in its interpolation. This method was first seen in actuarial literature in the article *Extrapolating, Smoothing, and Interpolating Development Factors* by Sherman, and later in *Analysis of Loss Development Patterns Using Infinitely Decomposable Percent of Ultimate Curves* by Robbin and Homer.

The key to this approach is in handling partial periods, e.g., the less than 12-month factor for an accident year period or less than 24-month factor for a policy year period. The Average Date of Loss Approximation formula takes into account two adjustments. The first adjustment accounts for the unearned exposure of the partial period, and the second adjusts for the maturity of a partial period.

Exposure adjustment

The first adjustment accounts for the unearned exposure of the partial periods. For example, your 3-month to ultimate factor could project a full accident year ultimate or a partial accident year ultimate. If you want to project a full year ultimate, you need to multiply your 3-month to ultimate factor by a factor of 12/3 or 4 to account for the remaining 9 months of exposure. Or another way to look at it is to divide your resulting interpolated age-to-ultimate factor by your cumulative exposure, defined as $G(x)$.

For an accident period:

$$G(x) = \text{MIN} \left(\frac{x}{k_{exp}}, 1 \right)$$

where k_{exp} = length of the exposure period, in months

For a policy period:

$$G(x) = \begin{cases} \frac{1}{2} * \text{MIN} \left(\frac{x}{k_{exp}}, 1 \right)^2, & \text{for } x \leq k_{exp} \\ 1 - \frac{1}{2} * \text{MAX} \left(2 - \frac{x}{k_{exp}}, 0 \right)^2, & \text{for } x > k_{exp} \end{cases}$$

Maturity adjustment

The second adjustment accounts for the maturity of a partial period. A partial period evaluated at 3 months cannot be assumed to have the same maturity as a full period evaluated at 3 months. For example, maturity at the end of a 3 months partial period is equivalent to a 7.5-month maturity for a full year. Therefore, the development factor to ultimate for 3 months should be the same as the 7.5-month to ultimate factor for 12 months.

Another way to look at this is to calculate the difference between the evaluation date and the average accident date for each development factor you have, defined by $M(x)$.

For an accident period:

$$M(x) = \text{MAX} \left(\frac{x}{2}, x - \frac{k_{exp}}{2} \right)$$

For a policy period:

$$M(x) = \begin{cases} \text{MAX}\left(\frac{x}{3}, x - 2 * \frac{k_{exp}}{3}\right), & \text{for } x \leq k_{exp} \\ \frac{\left((x - k_{exp}) + \left(\frac{1}{3}\right)(2 * k_{exp} - x)(1 - G(x))\right)}{G(x)}, & \text{for } x > k_{exp} \end{cases}$$

SAMPLE INTERPOLATION PROCEDURE

The easiest way to walk through the steps of the procedure is to provide an example.

The data we will use is:

Age	12-24	24-36	36-48	48-60	60-72	72-84	84-Ult
Selected	2.110	1.350	1.105	1.080	1.034	1.034	1.074
Cumulative	3.903	1.850	1.370	1.240	1.148	1.111	1.074
Ratio to Ultimate	0.256	0.541	0.730	0.806	0.871	0.900	0.931

We also assume the exposure periods are accident years ($k_{exp} = 12$). Our objective is to interpolate these development factors to be applicable to development ages {3,15,27,39}.

Step 1: Identify the array of known_x's, adjusted for maturity

For an array of known development ages and a given exposure period length, the maturity adjustment $M(x)$ calculates the difference between the evaluation date and average accident date for each development factor that you have.

$$known_x = \{12, 24, 36, 48, 60, 72, 84\}$$

$$known_x^{adj} = M(known_x) = \{6, 18, 30, 42, 54, 66, 78\}$$

Step 2: Identify the array of known_y's

The known_y's can be chosen from among the cumulative development factors or ratio to ultimate factors. For our example, we use ratio to ultimates

$$known_y = \{0.256, 0.541, 0.730, 0.806, 0.871, 0.900, 0.931\}$$

Step 3: Identify array of new_x's, adjusted for maturity

As with the known development ages, the new (target) development ages must also be adjusted for maturity:

$$new_x = \{3, 15, 27, 39, 51, 63, 75\}$$

$$new_x^{adj} = M(new_x) = \{1.5, 9, 21, 33, 45, 57, 69\}$$

Step 4: Find the 2 closest known_y's

For each new_x^{adj} , find the two closest known data points that border each side of the development factors you want. If you are extrapolating past the last development factor, use the last two factors, and if you are extrapolating to ages earlier than you have data, use the first two factors. You use these factors for the interpolation. For example:

$$new_{x_1}^{adj} = 1.5, \text{ with closest 2 known } x, y \text{ pairs being } \{6, 0.256\} \text{ and } \{18, 0.541\}$$

$$new_{x_2}^{adj} = 9, \text{ with closest 2 known } x, y \text{ pairs being } \{6, 0.256\} \text{ and } \{18, 0.541\}$$

$$new_{x_3}^{adj} = 21, \text{ with closest 2 known } x, y \text{ pairs being } \{18, 0.541\} \text{ and } \{30, 0.730\}$$

Step 5: Adjust the known_y's for cumulative exposure

The next step is to adjust the development factors that you have for the amount of cumulative exposure. To accomplish this:

- If using cumulative development factors, multiply each of the factors by their cumulative exposure $G(x)$
- If using the ratio to ultimates, divide each of the factors by their cumulative exposure $G(x)$

Since the known factors in this example begin at 12 months, none of the known_y's need to be adjusted, as the cumulative exposure is:

$$G(x) = \min\left(\frac{x}{12}, 1\right) = 1$$

Step 6: Apply an interpolation formula to calculate interpolated factors

As described above, Arius allows you to choose among four interpolation functions: Exponential, Inverse Power, Linear, and Weibull. For each function, the approach to arrive at the new interpolated factors is the same. First, calculate an intercept (a) and slope (b) given the two adjusted data points using transformed linear equations from the previous section. Then, plug the parameters back into the original equations to arrive at a new_y.

For example, to calculate the first interpolated value under each of the above algorithms, we have the following:

$$x_1 = 6, x_2 = 18, y_1 = 0.256, y_2 = 0.541$$

Linear:

Solving the linear equations system from the last column of the table in the previous section returns $a = 0.114$; $b = 0.024$. From this,

$$y_1^{adj} = a + b * new_{x_1}^{adj} = 0.114 + 0.024 * 1.5 = 0.150$$

Exponential:

Along with the above values, find $\ln(y_1) = -1.363$; $\ln(y_2) = -0.164$. Then, solving the linear equations system from the last column of the table in the previous section returns $a = -1.735$; $b = 0.062$. From this,

$$y_1^{adj} = \exp(a + b * new_{x_1}^{adj}) = \exp(-1.735 + 0.062 * 1.5) = 0.194$$

Inverse Power:

Along with the above values, find $\ln\left(\frac{1}{x_1}\right) = -1.792$, $\ln\left(\frac{1}{y_1-1}\right) = 1.066$, $\ln\left(\frac{1}{x_2}\right) = -2.890$, and $\ln\left(\frac{1}{y_2-1}\right) = 0.163$. Solving the linear equations returns $a = 3.069$; $b = 1.118$. From this,

$$y_1^{adj} = \left(1 + \exp(a) * new_{x_1}^{adj-b}\right)^{-1} = (1 + \exp(3.069) * 1.5^{-1.118})^{-1} = 0.068$$

Weibull:

Along with the figures calculated above, find $\ln(-\ln(1 - y_1)) = -1.218$ and $\ln(-\ln(1 - y_2)) = -0.250$. Solving the linear equations returns $a = -2.793$; $b = 0.879$. From this,

$$y_1^{adj} = 1 - \exp\left(-e^a new_{x_1}^{adjb}\right) = 1 - \exp\left(-e^{-2.793} * 1.5^{0.879}\right) = 0.083$$

Step 7: Adjust the new_y's for cumulative exposure

Up to this point, the interpolated factor from age x-to-ultimate is only projecting the partial 3-month period ultimate for this example. The final step is to adjust the development factors for the amount of cumulative exposure.

To accomplish this:

- If using cumulative development factors, divide each of the factors by their cumulative exposure $G(x)$
- If using the ratio to ultimates, multiply each of the factors by their cumulative exposure $G(x)$

For this example:

$$G(x_1) = \min\left(\frac{new_{x_1}}{12}, 1\right) = \min\left(\frac{3}{12}, 1\right) = 0.250$$

So, the adjusted new_y's for the first period (derived in step 6) are calculated as follows:

Linear:	$y = 0.150 * 0.250 = 0.038$
Exponential:	$y = 0.194 * 0.250 = 0.049$
Inverse Power:	$y = 0.068 * 0.250 = 0.017$
Weibull:	$y = 0.083 * 0.250 = 0.021$

These factors are then converted either to incremental selected development factors for display on exhibits of Type = {Development, Other}, or to Ratio-to-Ultimate factors for display on exhibits of Type = {Ratio, Average}.

3. Use Case #1: Interim Reserve Analyses

In many cases, you may find it beneficial to perform an interim analysis where you append a partial diagonal of data and use your prior selected development factors (after interpolation) to project ultimate losses.

For example, suppose you perform a full analysis at third quarter using triangles evaluated as of 9/30, and selecting development factors with development ages 9, 21, 33, and so on. In this case, you may wish to append a 3-month diagonal to perform a quick roll-forward analysis as of 12/31 without needing to reselect your development factors.

Within Arius, you accomplish this by interpolating your 9-21-33 development factors to 12-24-36 development factors. These interpolated factors are then automatically applied to the data evaluated at 12/31 to derive new ultimate projections.

When interpolating selected development factors for interim reserve analyses, Arius uses the interpolation functions and algorithms described in the previous section.

For example, given the following arrays:

Age	9-21	21-33	33-45	45-57	57-69	69-81	81-Ult
Selected	2.350	1.587	1.170	1.088	1.046	1.034	1.083
Cumulative	5.556	2.364	1.490	1.274	1.171	1.120	1.083
Ratio to Ultimate	0.180	0.423	0.671	0.785	0.854	0.893	0.923

Arius calculates an array of new y's as either cumulative factors or ratio to ultimate values and then converts these values back to incremental development factors for display on the exhibit:

Age	12-24	24-36	36-48	48-60	60-72	72-Ult
Linear - Cumulative	1.822	1.495	1.151	1.078	1.043	1.111
Exponential - Cumulative	1.824	1.471	1.149	1.077	1.043	1.111
Inv Power - Cumulative	1.790	1.423	1.143	1.075	1.042	1.109
Weibull - Cumulative	1.801	1.409	1.141	1.074	1.042	1.109
Linear – Ratio to Ultimate	1.823	1.443	1.147	1.077	1.043	1.110
Exponential – Ratio to Ultimate	1.824	1.471	1.149	1.077	1.043	1.111
Inv Power –Ratio to Ultimate	1.801	1.409	1.141	1.074	1.042	1.109
Weibull – Ratio to Ultimate	1.790	1.423	1.143	1.075	1.042	1.109

Interim reserve analysis can be performed within Arius following these steps.

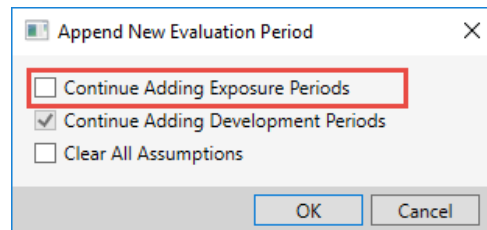
1. Prepare your project for interpolation.
2. Define which interpolated results display on exhibits.
3. Select interpolated development factors.
4. Project method ultimates.

PREPARE YOUR PROJECT FOR INTERPOLATION

Development factor interpolation is activated automatically within Arius based on the project's data structure. Naturally, the first step is to set up your project to include a partial diagonal.

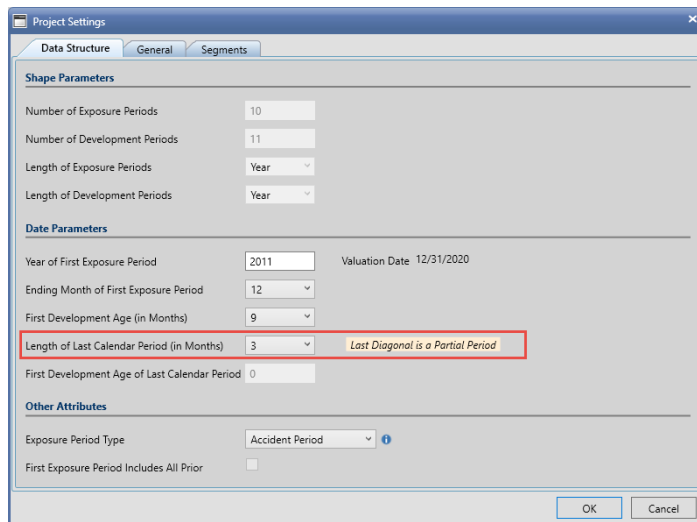
A partial last diagonal occurs when the last calendar period's length is less than the triangles' development periods. In the steps below, we use the above example where an actuary rolls forward a 9/30 year x year analysis to a 12/31 year x year analysis by appending a partial diagonal with a length of 3 months.

1. Append a new diagonal to your 9/30 Arius project using MODIFY STRUCTURE | APPEND NEW EVALUATION PERIOD. If you do not want to add a new exposure period, uncheck the box to **Continue Adding Exposure Periods**.



2. Under the PROJECT SETTINGS | DATA STRUCTURE tab, modify the **Length of Last Calendar Period (in Months)** parameter to the number of months included in the latest evaluation period (e.g., 3 months).

When the number of months entered here indicates that this evaluation is shorter than the full development period, the note **Last Diagonal is a Partial Period** displays, indicating that interpolation capabilities are active for this Arius project. Verify your entry by reviewing the calculated **Valuation Date** displayed on this screen.



3. Select **OK** and load data as of the Valuation Date.

- Select the collection **Loss Amounts** found under the DATA | INPUTS folder and open **Data #16 Paid Loss**. Note the last diagonal represents the partial 3-month diagonal.

COMMAUTOBI > Data > Paid Loss

Paid Loss - Cumulative

Accident Year	9	21	33	45	57	69	81	93	105	117	120
12-2011	5,825	96,385	188,044	297,746	297,746	297,746	297,746	387,746	387,746	415,746	415,746
12-2012	1,050	112,355	314,084	316,745	389,245	412,972	412,972	438,472	438,472	438,472	
12-2013	23,250	628,524	683,524	869,047	869,047	930,736	1,605,047	1,605,047	1,605,047		
12-2014	26,654	171,932	335,602	431,712	596,712	636,712	636,712	636,712			
12-2015	3,709	152,467	323,669	358,224	499,777	499,777	512,277				
12-2016	6,700	25,089	178,624	327,124	406,824	406,824					
12-2017	700	82,445	193,544	193,544							
12-2018	9,594	80,214	104,481								
12-2019	11,585	41,005									
12-2020	500	10,694									

- Select the collection **Paid Loss Development** found under the DETERMINISTIC | ANALYSIS | LOSSES folder and open Exhibit #40 **Paid Loss Development**. Note the last diagonal of development factors is suppressed. Below the selected development factors, there is an interpolation section, where you select interpolated factors at 12-24-36 months.

COMMAUTOBI > Exhibits > Paid Loss Development

Paid Loss Development

Accident Year	9-21	21-33	33-45	45-57	57-69	69-81	81-93	93-105	105-117	117-Ult
12-2012	107.005	2.795	1.008	1.229	1.061	1.000	1.062	1.000		
12-2013	27.033	1.088	1.271	1.000	1.071	1.724	1.000			
12-2014	6.451	1.952	1.286	1.382	1.067	1.000				
12-2015	41.107	2.123	1.107	1.395	1.000					
12-2016	3.745	7.120	1.831	1.244						
12-2017	117.778	2.348	1.000							
12-2018	8.361	1.303								
12-2019	3.539									
12-2020										
Vol Wtd Avg	15.545	1.703	1.230	1.199	1.053	1.340	1.013	1.000		
7 Yr Vol Wtd Avg	14.377	1.703	1.230	1.199	1.053	1.340	1.013	1.000		
5 Yr Vol Wtd Avg	11.807	2.218	1.271	1.199	1.053	1.340	1.013	1.000		
5 Yr Vol Wtd Avg Exc Hi/Lo	12.887	2.096	1.235	1.295	1.065	1.000				
3 Yr Vol Wtd Avg	9.309	2.539	1.263	1.346	1.052	1.340	1.013	1.000		
Selections at 9/30/2014	5.723	1.832	1.399	1.323	1.036	1.430	1.013	1.000	1.000	1.000
Inv Power	10.682	2.209	1.399	1.186	1.104	1.065	1.044	1.031	1.023	1.084
Exponential	5.068	2.720	1.727	1.307	1.130	1.055	1.023	1.010	1.004	1.003
Weibull	7.789	2.595	1.625	1.281	1.130	1.060	1.027	1.011	1.005	1.003
Default	9.309	2.539	1.263	1.346	1.052	1.340	1.009	1.006	1.003	1.003
Manual Selected										
Selected	9.309	2.539	1.263	1.346	1.052	1.340	1.009	1.006	1.003	1.003
Cumulative	57.779	6.207	2.445	1.936	1.438	1.368	1.020	1.011	1.006	1.003
Ratio to Ultimate	0.017	0.161	0.409	0.517	0.695	0.731	0.980	0.989	0.994	0.997
Interpolated Ages	BLNK-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-Ult
Default										
Manual Selected										
Selected Interpolated										
Cumulative Interpolated										
Ratio to Ultimate Interpolated										



Note:

Arius will automatically hide the resulting age-to-age factors for a partial last diagonal.



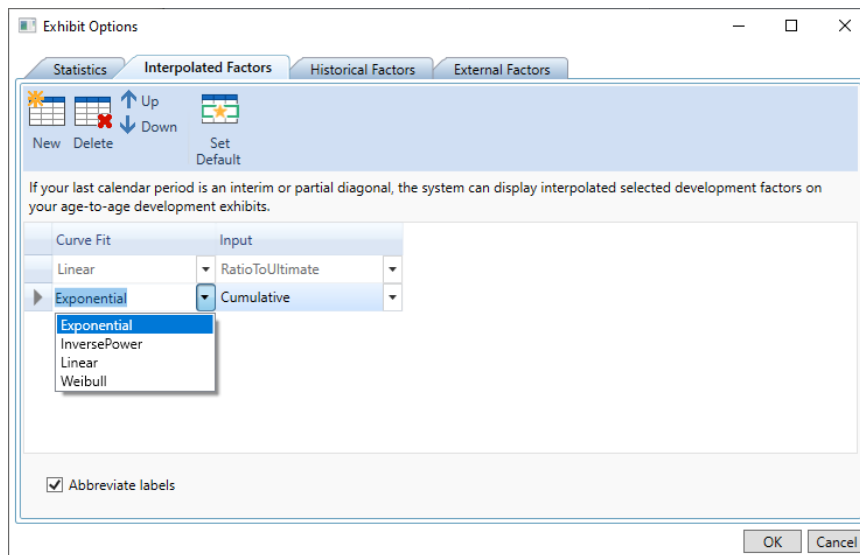
Note:

The interpolated ages are derived by the development age labels (Length of Development Period – Length of Last Calendar Period).

DEFINE WHICH INTERPOLATED RESULTS DISPLAY ON EXHIBITS

Within Arius, you have the option to display interpolated factors using four different curve fits (Linear, Exponential, Weibull, Inverse Power) and two different inputs (Cumulative, Ratio to Ultimate).

1. To select which interpolated factors to display, go to **EXHIBIT OPTIONS | INTERPOLATED FACTORS**.



Note:

You can use the buttons at the top of the window to add new interpolated results, delete existing ones from the list, and rearrange the list as desired.

2. Click the **NEW** button and select from the drop-down button within a row to modify the desired curve fit and input.
 - **Curve Fit** – This drop-down list selector identifies the type of curve to use.
 - **Input** – This drop-down list selector identifies the input values to use.
 - **Abbreviate Labels** – Check this box if you want to display abbreviated labels to the left of your interpolation calculations on your exhibits.
3. Click **OK** to save your preferences. When **PROJECT SETTINGS | LENGTH OF LAST CALENDAR PERIOD (MONTHS)** is a partial period, the interpolated results automatically appear in the interpolation block on all exhibits with selected factors.

SELECT INTERPOLATED DEVELOPMENT FACTORS

Similar to the standard development factor selections above, you can set defaults or manually select interpolated development factors.

1. Select the collection **Paid Loss Development** found under the DETERMINISTIC | ANALYSIS | LOSSES folder and open Exhibit **#40 Paid Loss Development**. Note the resulting interpolated factors are now displayed.

COMMAUTOBI > Exhibits > Paid Loss Development											
Paid Loss Development											
Accident Year	9-21	21-33	33-45	45-57	57-69	69-81	81-93	93-105	105-117	117-129	129-Ult
12-2011	16.547	1.951	1.583	1.000	1.000	1.000	1.302	1.000	1.072		
12-2012	107.005	2.795	1.008	1.229	1.061	1.000	1.062	1.000			
12-2013	27.033	1.088	1.329	1.256	1.186	1.121	1.059				
12-2014	6.451	1.952	1.286	1.382	1.067	1.000					
12-2015	41.107	2.123	1.107	1.395	1.000						
12-2016	3.745	7.120	1.831	1.244							
12-2017	117.778	2.348	1.000								
12-2018	8.361	1.303									
12-2019	3.539										
12-2020											
Vol Wtd Avg	15.611	1.720	1.278	1.262	1.094	1.060	1.092	1.000	1.072		
7 Yr Vol Wtd Avg	14.377	1.703	1.278	1.262	1.094	1.060	1.092	1.000	1.072		
5 Yr Vol Wtd Avg	11.807	2.218	1.294	1.295	1.094	1.060	1.092	1.000	1.072		
5 Yr Vol Wtd Avg Exc Hi/Lo	12.887	2.096	1.265	1.286	1.043	1.000	1.062	1.000			
3 Yr Vol Wtd Avg	9.309	2.539	1.263	1.346	1.113	1.068	1.092	1.000	1.072		
Selections at 9/30/2013	9.309	2.539	1.263	1.346	1.116	1.092	1.068	1.045	1.022	1.022	1.166
Interpolated from 3/31/2013	2.889	1.890	1.325	1.173	1.137	1.105	1.074	1.044	1.029	1.028	1.330
Inv Power	8.022	2.225	1.483	1.255	1.157	1.106	1.076	1.057	1.044	1.036	1.151
Exponential	3.211	2.269	1.729	1.419	1.240	1.138	1.079	1.046	1.026	1.015	1.020
Weibull	6.784	2.577	1.691	1.350	1.187	1.102	1.055	1.030	1.016	1.008	1.008
Default	9.309	2.539	1.263	1.346	1.116	1.092	1.068	1.045	1.022	1.022	1.166
Manual Selected											
Selected	9.309	2.539	1.263	1.346	1.116	1.092	1.068	1.045	1.022	1.022	1.166
Cumulative	66.583	7.153	2.817	2.231	1.658	1.485	1.360	1.273	1.218	1.192	1.166
Ratio to Ultimate	0.015	0.140	0.355	0.448	0.603	0.673	0.735	0.786	0.821	0.839	0.858
Interpolated Ages	BLNK-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
Linear - Ratio to Ultimate		5.213	1.954	1.288	1.275	1.110	1.086	1.062	1.039	1.022	1.185
Exponential - Ratio to Ultimate		6.676	2.132	1.283	1.284	1.110	1.086	1.062	1.039	1.022	1.185
Weibull - Ratio to Ultimate		5.886	2.009	1.287	1.272	1.109	1.085	1.062	1.039	1.022	1.185
Inverse Power - Ratio to Ultimate		5.909	1.986	1.290	1.268	1.109	1.085	1.061	1.038	1.022	1.185
Default		5.213	1.954	1.288	1.275	1.110	1.086	1.062	1.039	1.022	1.185
Manual Selected											
Selected Interpolated		5.213	1.954	1.288	1.275	1.110	1.086	1.062	1.039	1.022	1.185
Cumulative Interpolated		26.928	5.166	2.644	2.053	1.611	1.451	1.337	1.259	1.211	1.185
Ratio to Ultimate Interpolated		0.037	0.194	0.378	0.487	0.621	0.689	0.748	0.794	0.826	0.844



Note:
Verify that the factors in your Selected Row match your 9/30 factor selections, or alternatively you can default to the historical selections labeled **Selections at 9/30**.

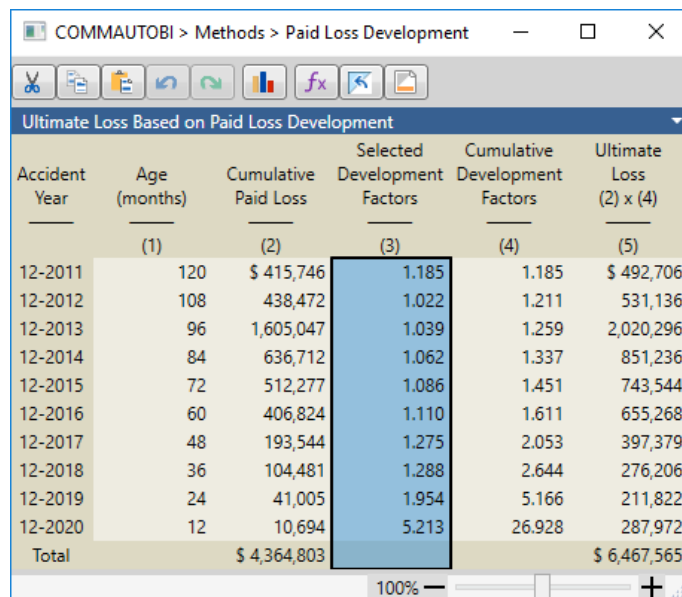
2. Set default interpolated development factor selections by right-clicking on a cell, group of cells, or row label and choosing **DEFAULT | SET AS DEFAULT**. You can also set a default across all segments for specified exhibits using the **DEFAULTS | EXHIBITS** option from the Home ribbon. For more information, refer to the document *Default Settings to Automate Analysis* found under **HELP | USER DOCUMENTATION**.

- Manual selections can be entered in the **Manual Selected** row (or right-click a statistic and select **Copy Value to Manual Selected**).
- Once interpolated development factor selections are made, Arius methods using development factors reference the selected interpolated development factors. See the following section for more information.


PROJECT METHOD ULTIMATES

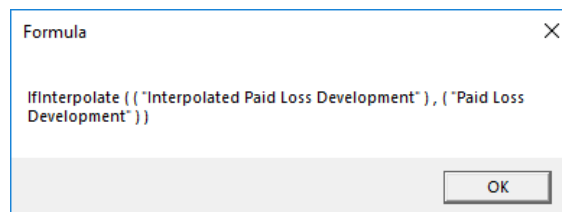
When using a partial last diagonal, all methods that utilize development factor selections automatically use the interpolated factor selections from the appropriate exhibit via the **Ifinterpolate** function.

- Select the collection **Paid Loss Development** found under the DETERMINISTIC | ANALYSIS | LOSSES folder and open Method **#1 Paid Loss Development**. Highlight column (3), the **Selected Development Factors**.



Accident Year	Age (months)	Cumulative Paid Loss	Selected Development Factors	Cumulative Development Factors	Ultimate Loss (2) x (4)
	(1)	(2)	(3)	(4)	(5)
12-2011	120	\$ 415,746	1.185	1.185	\$ 492,706
12-2012	108	438,472	1.022	1.211	531,136
12-2013	96	1,605,047	1.039	1.259	2,020,296
12-2014	84	636,712	1.062	1.337	851,236
12-2015	72	512,277	1.086	1.451	743,544
12-2016	60	406,824	1.110	1.611	655,268
12-2017	48	193,544	1.275	2.053	397,379
12-2018	36	104,481	1.288	2.644	276,206
12-2019	24	41,005	1.954	5.166	211,822
12-2020	12	10,694	5.213	26.928	287,972
Total		\$ 4,364,803			\$ 6,467,565

- Click the **Show Formula**  button. This formula relies on the **Ifinterpolate** function, which returns the **Interpolated Selected** row of factors if the partial period flag = TRUE; otherwise, it returns the **Selected** row of factors. The factors should always match the development age shown in column (1).



Formula

Ifinterpolate (("Interpolated Paid Loss Development") , ("Paid Loss Development"))

OK

Note the Ultimate Loss in column (5) always represents the full-period ultimate loss (e.g., 12-month ultimate loss in the example above). Any required proration is performed after selecting full-period ultimate losses, using the **Selected Proportion Earned Array**.



Note:

When building user-defined methods, be sure to utilize the **Ifinterpolate** function so that these methods can be used appropriately for interim reserve analyses.

4. Use Case #2: Interpolating Historical and External Factors

The Arius interpolation algorithms described in section 2 are also used to interpolate and extrapolate historical and external factors for display on exhibits. As new evaluation periods are appended to the Arius project file, Arius automatically *extrapolates* any historical or external factors. Similarly, if periods are removed, Arius automatically *compresses* the tail factors.

Also, you have the option to *interpolate* any historical or external factor set. To do this, select the **Interpolate** checkbox, found under the EXHIBIT OPTIONS | HISTORICAL FACTORS or EXHIBIT OPTIONS | EXTERNAL FACTORS dialogs. Interpolation and extrapolation of historical and external factors are performed using the curve fit and input parameters set via the **Set Default** button found on the EXHIBIT OPTIONS | INTERPOLATED FACTORS tab.

For more information on displaying these benchmarks on exhibits, see *Displaying Historical Factors on Exhibits* and the *Displaying External Factors on Exhibits* documents found in Arius under HELP | USER DOCUMENTATION. For example, given the following array of selected development factors at 12/31/xx:

Age	12-24	24-36	36-48	48-60	60-72	72-84	84-Ult
Selected	2.110	1.350	1.105	1.080	1.034	1.034	1.074
Cumulative	3.903	1.850	1.370	1.240	1.148	1.111	1.074
Ratio to Ultimate	0.256	0.541	0.730	0.806	0.871	0.900	0.931

If you append a new evaluation period (MODIFY STRUCTURE | APPEND NEW EVALUATION PERIOD), selected factors at 12/31/xx would be saved to the Historical Factor Library. Arius automatically extrapolates the 84-ultimate factor on the development factor exhibit.

Assuming you are using the Arius default of Curve Fit = Exponential and Input = Ratio to Ultimate, you will see:


Age	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-Ult
Selected at 12/31/xx	2.110	1.350	1.105	1.080	1.034	1.034	1.034	1.039

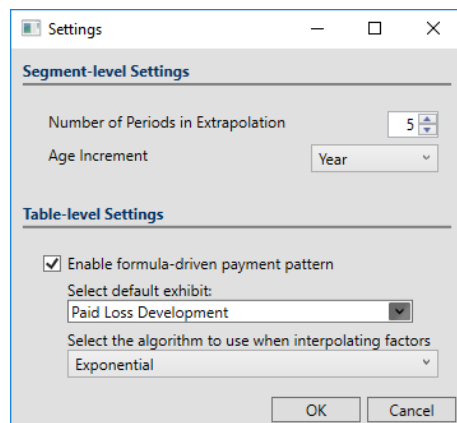
Note that when extrapolating to the 84-96 period to return the 1.034 value, the interpolation algorithm fits a curve through the closest two data points {72-84, 0.900} and {84-ult, 0.931} to determine the slope (*b*) and intercept (*a*). The resulting 96-Ult factor is then derived by taking $1.074/1.034 = 1.039$ to ensure cumulative development from 84-ultimate is identical for both evaluations.

If you modify the data structure by selecting PROJECT SETTINGS | DATA STRUCTURE and setting the **First Development Age** = 3, you can select the option to interpolate the historical factor set. When selected, Arius shows interpolated historical factors on all relevant exhibits. Continuing to assume you are using the system default of Curve Fit = Exponential and Input = Ratio to Ultimate, you see:

Age	3-15	15-27	27-39	39-51	51-63	63-75	75-87	87-Ult
Interpolated from 12/31/xx	6.739	1.887	1.284	1.099	1.068	1.034	1.034	1.068

5. Use Case #3: Interpolating Payment Patterns

The Arius interpolation algorithms are used when interpolating and extrapolating payment patterns for cash flow reports. The built-in payment pattern arrays include data assumption #801 **Loss Payment Pattern**, #476 **ALAE Payment Pattern**, #477 **Salvage and Subrogation Payment Pattern**, and #125 **ULAE Payment Pattern**. These arrays are special resizable row type and contain several features that differentiate them from standard row arrays. For example, the **Settings**  button allows you to set the number of periods to extrapolate and to modify the resulting cash flow reports' age increment.



If the **Enable formula-driven payment pattern** option is checked, Arius automatically interpolates and extrapolates the **Ratio to Ultimate** values from the selected exhibit (e.g., Paid Loss Development) using the selected curve fit (e.g., Exponential).

For example, if you set the Number of Extrapolation periods = 5, Arius *extrapolates* the ratio to ultimate pattern an additional 5 periods. If you set the Age Increment = Quarters, Arius interpolates the ratio to ultimate pattern to quarterly intervals to produce cash flows at this more granular level.

For more information on payment patterns and how they are used in Arius Cash Flow reporting, see the *Cash Flow Reports* document found in Arius under **HELP | USER DOCUMENTATION**.

For example, given the following array of selected development factors at 12/31/xx:

Age	12-24	24-36	36-48	48-60	60-72	72-84	84-Ult
Selected	2.110	1.350	1.105	1.080	1.034	1.034	1.074
Cumulative	3.903	1.850	1.370	1.240	1.148	1.111	1.074
Ratio to Ultimate	0.256	0.541	0.730	0.806	0.871	0.900	0.931

In the Loss Payment Pattern object, under the payment pattern **Settings** dialog, set **Number of Periods in Extrapolation** = 2, **Age Increment** = Year, **Enable Formula-driven Payment Pattern** = TRUE, and **Algorithm** = Exponential. Arius would return the following payment pattern:

Age	12	24	36	48	60	72	84	96	108
Payment Pattern	0.256	0.541	0.730	0.806	0.871	0.900	0.931	0.963	1.000

Note that when extrapolating to the 96th period (to arrive at the 0.963), the interpolation algorithm fits a curve through the closest two data points {72-84, 0.900} and {84-Ult, 0.931}, to determine the slope (b) and intercept (a).

The payment pattern's last entry is forced to 1.000 to ensure all indicated unpaid values are paid out in the cash flow reports.

If you now modify the **Age Increment** = Quarter and adjust the **Number of Periods in Extrapolation** = 8 to equate to the 2 years above, Arius would return the following payment pattern:

Age	12	15	18	21	24	27	108
Interpolated from 12/31/xx	0.256	0.309	0.372	0.449	0.541	0.583	1.000

For more information on how these factors were derived, see the section entitled *Details behind the Calculations*. Note that for payment patterns, there is no need to adjust for unearned exposures so Steps 5 and 7 are omitted.