

## OVERVIEW

In certain scenarios smoothing can result in a more stable reserve estimate, such as age-to-age factor triangles where there is an erratic development pattern. Smoothing is also useful when applied where data is limited or unreliable.

In any of your Arius exhibits you can apply smoothing using a **linear** or an **exponential** smoothing algorithm. Either smoothing algorithm can be applied to triangles at the top of your exhibits or to the Default row on exhibits with **Selected** rows.

## HOW TO APPLY SMOOTHING

Note that smoothing can be applied to any type of exhibit (e.g., development, ratio, average, other).

1. In your exhibit select two or more contiguous factors on a row in the calculated triangle or in the Default row.
  - Note that the Default row tail factor/final cell cannot be selected for smoothing.
2. Right-click on these factors and choose **Smoothing**, then choose **Smooth Linear** or **Smooth Exponential**.
  - If your selection includes the final cell of the Default row, less than 2 factors, or cells that are not from the calculated triangle or Default row, then these choices will appear greyed-out and will be unavailable.
3. Purple borders are displayed around the set of smoothed factors.
  - When smoothing factors in the exhibit's triangle, these smoothed factors are used in the calculation of applicable statistics rows on the exhibit.
  - When smoothing factors in the Default row, these factors will become your **Selected** factors (unless an over-riding factor is entered into the Manual Selected row).

### Example 1: Linear smoothing - applied to age-to-age factors

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
2011	1.6633	1.2568	1.1558	1.0627	1.0348	1.0196	0.9987	0.9996	0.9998	
2012	1.7333	1.1980	1.1194	1.0566	1.0426	1.0038				
2013	1.7892	1.1839	1.1781	1.0822	1.0192	1.0091				
2014	1.8929	1.2366	1.1358	1.0147	1.0375	1.0198				
2015	2.0762	1.1624	1.1152	1.0731	1.0348					
2016	1.5963	1.2274	1.1435	1.0556						
2017	1.5922	1.2225	1.1209							
2018	1.8060	1.2026								
2019	1.8130									
2020										
Volume Weighted Average	1.7595	1.2142	1.1416	1.0595	1.0338	1.0133				

### Results:

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
2011	1.6633	1.2568	1.1558	1.0627	1.0348	1.0044	1.0044	1.0044	1.0044	
2012	1.7333	1.1980	1.1194	1.0566	1.0426	1.0038	1.0001	1.0002		
2013	1.7892	1.1839	1.1781	1.0822	1.0192	1.0091	1.0281			
2014	1.8929	1.2366	1.1358	1.0147	1.0375	1.0198				

## Example 2: Exponential smoothing - applied to Default row factors

Sample P&C Industry - Oth_Liab_Occ	1.7315	1.3212	1.1739	1.1009	1.0568	1.0372	1.0274	1.0179	1.0136	1.0804
Inverse Power Curve	2.3413	1.4412	1.2302	1.1451	1.1014	1.0757	1.0591	1.0477	1.0395	1.2193
Exponential Curve	1.7718	1.4967	1.3197	1.2058	1.1324	1.0852	1.0548	1.0353	1.0227	1.0412
Weibull Curve	2.0158	1.4754	1.2802	1.1819	1.1246	1.0884	1.0642	1.0475	1.0357	1.1170
Default	1.6972	1.6987	1.4853	1.1721	1.1495	1.0660	1.0128	1.0000	1.0688	1.0412
Manual Selected										
Selected	1.6972	1.6987	1.4853	1.1721	1.1495	1.0660				
Cumulative	6.9324	4.0846	2.4045	1.6189	1.3812	1.2015				
Ratio to Ultimate	0.1443	0.2448	0.4159	0.6177	0.7240	0.8323				

  

Smooth Linear	Set as Default
Smooth Exponential	Remove as Default
Remove Smoothing	Copy Value To Manual Selected
	Exclude Factor(s) from Statistics
	Include Factor(s) in Statistics
	Smoothing
	Show Formula
	Source Data

### Results:

Weibull Curve	2.0158	1.4754	1.2802	1.1819	1.1246	1.0884	1.0642	1.0475	1.0357	1.1170
Default	1.6972	1.6987	1.4853	1.1721	1.1495	1.0590	1.0439	1.0291	1.0144	1.0412
Manual Selected										
Selected	1.6972	1.6987	1.4853	1.1721	1.1495	1.0590	1.0439	1.0291	1.0144	1.0412

## HOW TO REMOVE SMOOTHING

To remove smoothing from your factors, right-click on any factor in the set of smoothed factors within the purple border and choose **Remove Smoothing**. This will remove all smoothing from the set of contiguous smoothed factors and remove the purple border.

NOTE: If the structure of your file is modified to remove exposure and/or development periods, resulting in the removal of a factor or factors which were included in a contiguous set of smoothed factors, then smoothing is removed from the entire group of contiguous factors in the set.

## THE ALGORITHMS

### Linear Smoothing

$X = \text{Product of highlighted factors} = 1.0196 \times .9987 \times .9996 \times .9998 = 1.0177$

$N = \text{Number of highlighted factors} = 4$

$Z = \text{Replacement factors} = X^{(1/N)} = 1.0044$

### Exponential Smoothing

$X = \text{Product of highlighted factors} = 1.0660 \times 1.0128 \times 1.000 \times 1.0688 = 1.1539$

$N = \text{Number of highlighted factors} = 4$

$M = \text{Number of Interpolation Units} = N \times (N+1)/2 = 10$

$Z\_N = X^{(1/M)} = 1.0144$

$Z\_M = Z\_N \times Z\_M+1 \text{ (M from 1 to N-1)} = 1.0590 \ 1.0439 \ 1.0291 \ 1.0144$

These algorithms can also be understood as follows:

Let  $LDF_i$  represent the empirical (incremental) LDFs, and let  $LDF_i^s$  represent the smoothed (incremental) LDF for development period  $i$ . Suppose you want to smooth development periods  $i$  through  $i + n$ .

**Linear Smoothing:**

$$X = \prod_{j=i}^n LDF_j; \quad LDF_i^s = X^{\frac{1}{n}}$$

**Exponential Smoothing:**

$$X = \prod_{j=i}^n LDF_j; \quad M = n \times \frac{n+1}{2}; \quad LDF_{i+n}^s = X^{\frac{1}{M}}; \quad LDF_{i+n-1}^s = LDF_{i+n}^s * X^{\frac{1}{M}}$$

For all development periods  $k$  outside of  $i$  to  $i + n$ ,  $LDF_i^s = LDF_i$ .